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A thermal analysis of a cryogenic expander with a reciprocating dispacer/piston incorporating an annular regenerator on the displacer/piston outside diameter

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A THERMAL ANALYSIS OF A CRYOGENIC EXPANDER WITH A RECIPROCATING DISPLACER/PISTON INCORPORATING AN ANNULAR REGENERATOR ON THE DISPLACER/PISTON OUTSIDE DIAMETER

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CERTIFICATE OF APPROVAL

This thesis is accepted and approved in partial fulfillment of the requirements for the degree of Master of Science.

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Professor in Charge

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ABSTRACT

A cryogenic refrigerator using an expander with a reciprocating displacer or piston which has a thermal gradient from one end to the other suffers a loss of refrigeration due to heat transfer between the displacer/piston and the cylinder (shuttle heat transfer). This loss can be virtually eliminated by designing an annular thermal regenerator on the outside diameter of the displacer/piston in such a manner as to match the temperature profile of the displacer/piston and the cylinder wall throughout the cycle. The design parameters to accomplish this are analyzed to both determine the geometric configuration which will yield zero shuttle heat transfer and to predict the thermal performance of a device using this technique to reduce losses.

INTRODUCTION

A need for small amounts of refrigeration at cryogenic temperatures has resulted in the development of positive displacement reciprocating expanders. These expanders are operated on the Stirling cycle,[1] [2]Gifford-McMahon cycle,[3] [4]Solvay cycle,[5] Vuilliumier cycle[6] or some modification[7] of these cycles. Refrigeration is produced, in each of these cycles, by expanding compressed gas which has been precooled by a thermal regenerator. Heat from the thermal load is conducted through a thin cylinder wall rewarming the working gas to approximately the precooled temperature. The working gas is then vented through the thermal regenerator removing heat from the regenerator material, cooling the regenerator in preparation for the succeeding cycle.

There is a theoretical maximum refrigeration which can be produced from a given amount of compressed gas and a given pressure differential, which would all be available were it not for losses. This maximum refrigeration would occur if all of the compressed gas could be expanded over the total available pressure differential, but this cannot be accomplished for several reasons. Since the gas is cooled to the working temperature before it is expanded, a regenerative heat exchanger is used in all cycles. The regenerator introduces several losses; one, it must be pressurized and depressurized with each cycle consuming some of the gas before it can be expanded. This gas is all lost in the Gifford-McMahon and Solvay cycle expanders, however, in the Stirling and Vuilliumier cycle expanders there is some energy recovery in the regenerator gas when it is converted into kinetic energy on the expansion stroke. Two, the regenerator introduces a pressure drop as the gas passes through it thereby reducing the maximum available pressure differential over which the cold volume can expand the gas; and three, the regenerator is not a perfect heat exchanger so that the gas leaves the regenerator colder than it entered introducing a thermal load on the refrigerator. These losses are referred to as regenerator void volume loss, regenerator pressure drop loss and regenerator thermal inefficiency loss,[8] respectively. In addition to the losses introduced by the regenerator there are two other mechanisms which reduce the available refrigeration. Conduction through the cylinder and piston from the ambient to the cryogenic portion imposes a heat load on refrigeration and shuttle heat transfer,[9] a convective heat transfer process, transfers heat from the ambient to the cryogenic portion of the expander by alternately storing heat in the displacer/piston and the cylinder wall. This process occurs due to the displacer/piston and the cylinder wall having similar thermal profiles which move with respect to each other. Figure 1 illustrates the cylinder wall and displacer/piston temperature profiles at the extremes and midpoint of the displacer/piston motion. This motion causes the displacer/piston to be at a lower temperature than the cylinder during part of a cycle allowing heat to be transferred to the displacer/piston as illustrated in Figure 1a. The displacer/piston then moves and a condition occurs where the cylinder wall is at a lower temperature than the displacer/

piston so that heat now flows back into the cylinder as in Figure 1c. This process is repeated with each cycle pumping heat from the ambient temperature portion to the cold expansion volume portion of the expander.

The relative significance of each of the above mentioned losses is illustrated by the following table. This data was obtained through a design analysis of an optimized modified Solvay cycle refrigerator[10].

Refrigeration Allocation in an Optimized Modified Solvay Cycle Refrigerator

Loss due to regenerator void volume	34%
Loss due to regenerator pressure drop	9%
Loss due to regenerator inefficiency	11%
Loss due to cylinder conduction	1%
Loss due to shuttle heat transfer	12%
Net Refrigeration Available	33%
Theoretical Refrigeration	100%

While working on shuttle heat transfer Dr. Longsworth suggested a technique for arranging a regenerator in a cryogenic expander in such a way as to use the thermal cycling of the regenerator matrix material to reduce the shuttle heat transfer. This is accomplished by constructing an annular regenerator on the outside of the reciprocating displacer in a manner which eliminates the mismatch in temperature profile between the cylinder and the displacer/piston as the displacer/piston moves within the cylinder. This approach uses the temperature cycling of the regenerator, due to heat transfer between the regenerator and the working gas, to allow the displacer temperature profile to match the cylinder temperature profile established by conduction.

This paper develops and presents the parameters which are influential in the geometrical design of a regenerator which eliminates shuttle losses. It also presents a technique for analyzing the thermal performance of a cryogenic refrigerator employing the above designed regenerator. The technique is illustrated by using a computer program to predict the thermally optimum configuration for a Solvay cycle refrigerator operating at steady-state under typical conditions.

ANALYTICAL MODEL AND ASSUMPTIONS

The analytical model which is used to illustrate the geometry and develop the parameters necessary to eliminate shuttle heat transfer is presented in Figure 2. The construction of the expander is assumed to be a cylinder of low thermal conductivity stainless steel with an internal, reciprocating displacer/piston.

The displacer/piston has an annular regenerator on its external diameter. The high pressure working gas is introduced at the warm end of the cylinder and passes through the regenerator into the cold expansion volume. The expansion volume is swept from minimum to maximum volume by the displacer/piston motion with high pressure gas entering. The cycle timing is such that refrigeration is produced by expanding the gas to low pressure after the intake stroke. Part of the refrigeration will be consumed by refrigeration losses, the remainder will be referred to as net available refrigeration. The net available refrigeration can be used by conducting heat through the cylinder wall, from the heat load, to the working gas. The low pressure gas then passes back out through the regenerator to the warm end when it leaves the expander.

The temperature profile of the cylinder at steady-state condition is illustrated in Figure 3. The cylinder overall length is equal to the displacer/piston length plus the stroke. The thermal length is assumed equal to the displacer/piston length minus the stroke because a length of the cylinder equal to the stroke at the cold end of the cylinder is at the temperature of the cold volume, T_C, and an equal length at the warm end is at ambient temperature, T_a.

The displacer/piston temperature profile must match the cylinder profile for shuttle losses to be eliminated. This match in temperature profiles can be approached because the regenerator changes in temperature as the working fluid passes through it. The constant temperatures at the ends of the cylinder are also approached by the regenerator because an entrance portion of the regenerator at the end of a flow period reaches a constant temperature approximately equal to the entering gas temperature.

The technique which will allow the temperature profiles to be closely matched is illustrated in Figure 3. Assuming there was a thermal profile match with the displacer/piston at the warm end of the cylinder, when the displacer/piston moved to the cold end of the cylinder, a distance equal to the stroke, a thermal mismatch of magnitude, ΔT would occur if the displacer/piston were to maintain its original temperature profile, but the displacer/piston with an externally annular regenerator could change its temperature by the same amount ΔT if the regenerator were designed to accomplish this change.

The following assumptions are made to develop and analyze an expander which can accomplish the desired temperature change in the regenerator.

a. Regenerator pressure drop and thermal inefficiency are calculated assuming the regenerator is first pressurized then flow to and from the expansion volume occurs at a constant rate over a fixed percent of the cycle.

b. Friction factor in the pressure drop calculation is a function of the Reynolds Number only based on the flow computed above with the restriction that the flow is laminar (i.e. Reynolds Number <60).

c. The working fluid is helium gas which obeys the perfect gas law over the temperature range of interest.

d. The helium viscosity is a function of temperature alone.

e. The regenerator inefficiency is a function only of regenerator capacity ratio and NTU's.

f. The thermal mass of working gas which transfers heat with the regenerator is estimated to be the sum of all gas in the expansion volume plus a weighted average, based on temperature, of the gas in the regenerator void volume.

g. The cylinder conduction is based on conduction integrals for stainless steel.

h. The cylinder wall thickness is based on the maximum allowable wall stress.

i. The conduction through the displacer/piston can be made negligibly small with respect to the other losses.

j. Conduction through the regenerator is included in and adequately estimated by regenerator inefficiency data.

k. The Prandtl number is a constant over the conditions of interest.

l. The gas all passes through the regenerator (i.e. there is no gas leakage through the clearance).

m. The values of the temperature dependent properties, i.e. (k, μ, C_{pg}, C_{pr}) are calculated using linear temperature profiles over the field of interest.

GEOMETRIC AND THERMAL PARAMETERS FOR ZERO SHUTTLE HEAT TRANSFER

By matching the thermal profile of the regenerator with that of the cylinder wall as the regenerator moves with respect to the cylinder the shuttle heat transfer can essentially be reduced to zero. It is nearly possible to continuously match the thermal profiles of the annular regenerator and the cylinder wall since the temperature of a given point in the regenerator changes as the gas flow either adds or removes heat from the regenerator material. If this change in regenerator temperature is made to equal the temperature change of the adjacent cylinder wall corresponding to the regenerator's movement, the temperature difference can be held to essentially zero. By reference to Figure 3, the cylinder conduction length is displacer/piston length (L) minus the stroke (S), therefore, the slope of the temperature profile of the cylinder is $(T_a - T_c)/(L - S)$. The corresponding correct regenerator matrix temperature swing, ΔT, and stroke to cause zero shuttle losses is

$$\frac{\Delta T}{S} = \frac{T_a - T_c}{L - S} \qquad (1)$$

The parameters influencing the regenerator T are:

$$Q_r = M_r \cdot C_{pr} \cdot \Delta T \qquad (2)$$

$$Q_g = M_g \cdot C_{pg} \cdot (T_a - T_c). \qquad (3)$$

Since $Q_r = Q_g$ and

$$CR \equiv \frac{M_r \cdot C_{p\,r}}{M_g \cdot C_{p\,g}} \qquad (4)$$

defines a quantity referred to as capacity ratio, by substitution

$$S = \frac{L}{CR+1} \qquad (6)$$

Therefore, if the system is designed such that equation (6) is valid, it is assumed there will be no shuttle heat transfer.

THERMAL ANALYSIS

In order to perform a thermal analysis of a cryogenic expander the ideal refrigeration, QI, produced is computed then the thermal losses are determined and subtracted to predict the net available refrigeration. The following equations are used to predict the performance of a cryogenic refrigerator with an annular regenerator designed to give zero shuttle heat transfer.

Regenerator Void Volume

Regenerator void volume loss in the expander is based on the assumption that a given mass flow is compressed, but not all of the flow is expanded. The flow that is not expanded is used to pressurize and depressurize that regenerator, therefore, not producing refrigeration and introducing a loss.

The regenerator void volume loss is determined by

$$V_r = M_r/(\rho_{cu} \times (1.0 - p) \qquad (7)$$

where

$$M_r = CR \times C_{phe} \times (W/N)/C_{pr} \qquad (8)$$

The mass of gas used to pressurize and depressurize this volume each cycle is:

$$M_{he,h} = V_r \times p \times (\rho_{he,h} - \rho_{he,l}) \times N \qquad (9)$$

This mass of gas is subtracted from the compressor flow to determine the mass of gas which is expanded to produce the ideal refrigeration.

All other losses are converted to heat flows, and these losses are subtracted from the ideal refrigeration to yield the expected net refrigeration.

Ideal Refrigeration

The maximum ideal refrigeration which could be produced by a Gifford-McMahon cycle, Solvay cycle or modified Solvay cycle refrigerator can be computed by assuming that the mass of gas compressed minus the mass in the regenerator is expanded over the maximum available pressure differential. Figure 4 illustrates the ideal cycle diagram for these cycles. The pressure differential is considered as an independent variable. The expansion volume, V_c, can be calculated from the following equations

$$V_c = (W - M_{he,r})/(\rho_{he,vc} \times N) \tag{10}$$

The ideal refrigeration produced is therefore

$$QI = V_c \times (P_h - P_l) \times N \tag{11}$$

Equations for the Stirling cycle and Vuillimier cycle ideal refrigeration can be developed in a similar fashion.

Regenerator Pressure Drop Loss

Regenerator pressure drop, unlike the other thermal losses is not a heat leak but rather a reduction in the ideal refrigeration produced. This reduction reduces the available pressure differential over which the gas can be expanded. Figure 4 illustrates the effect of regenerator pressure drop.

To calculate this pressure drop an equation must be developed which takes the temperature gradient along the regenerator into consideration. This is accomplished by using the basic pressure drop equation:

$$\Delta P = \frac{G^2}{2\ gc}\ 4f\ \frac{L}{Dh} \quad \text{or} \quad \frac{KG^2f}{Dh} \tag{12}$$

Kays and London [11] present friction factors for layered screen type heat exchangers which indicate that the friction factor can be approximated by

$$f = \frac{64}{N_{RE}} \tag{13}$$

for 200 mesh screens with porosity of 0.67, if $N_{RE} < 60$. This type of layered screen regenerator construction is most common and is therefore considered here, but other constructions, i.e., packed spheres or other screen meshes, could be handled in a similar fashion. To account for variations in the Reynolds Number with changing temperature due to viscosity, the following expression can be used for the viscosity of helium gas over the temperature range of interest.

$$\mu = K_2\, T^{0.65} \tag{14}$$

yielding

$$f = \frac{K_3 T^{0.65}}{GD_h} \tag{15}$$

The effect of temperature on density can be determined using the perfect gas law (which helium obeys closely above 20°K).

$$\rho = \frac{P}{P_i}\frac{T_i}{T}\rho_i \qquad (16)$$

The pressure drop through the regenerator is held small by design, therefore the minimum pressure $P \geqslant .8P_i$ or $1 \geqslant P/P_i \geqslant 0.8$. The temperature change through the regenerator is large since the range 540°R to 140°R is assumed, thus $1 \leqslant T_i/T \leqslant 3.7$. By an order of magnitude analysis the pressure term is eliminated because of its relatively minor effect, yielding

$$\rho = \frac{T_i}{T}\rho_i \qquad (17)$$

Ackerman[12] indicates that the temperature profile is approximately linear over most of the regenerator length, therefore, the temperature at any position in the regenerator can be expressed as

$$T = T_i - \frac{T_i - T_o}{L} X \qquad (18)$$

Taking the derivative of equation (12) with respect to length and substituting (15), (17), (18).

$$\frac{dP}{dx} = \frac{K_4 G}{\rho_i T_i D_h^2}\left[T_i - \frac{T_i - T_o}{L} X\right]^{1.65} \qquad (19)$$

Integrating and substituting for ρ_i,

$$\Delta P = \frac{K_5 GL}{P_i D_h^2}\,\frac{T_i^{2.65} - T_o^{2.65}}{T_i - T_o} \qquad (20)$$

Once the pressure drop through the regenerator is defined the reduction in available refrigeration due to pressure drop is calculated from

$$Ql_{\Delta p} = V_c \times (\Delta P_h + \Delta P_l) \times N \tag{21}$$

Regenerator Thermal Inefficiency Loss

This loss is due to the gas not reaching complete thermal equilibrium with the regenerator. Ackerman[8] experimentally correlated this loss with the equation

$$Ql_{tin} = I_r \times M_{he,t} \times C_{phe} \times (T_a - T_c) \tag{22}$$

The thermal inefficiency data used is illustrated in Figure 5, obtained from Ackerman's[8] test results. The thermal mass of helium used in this equation is equal to the mass of gas in the expansion volume plus the mass of gas in the regenerator on a weighted average determined by the temperature difference which the gas experiences.

Conduction Losses

Conduction causes a loss of refrigeration by transferring heat from the ambient temperature end of the device to the cold end. There are three separate conduction paths to the cold end, the cylinder (almost always made of stainless steel because of its high strength to conductivity ratio), the regenerator, and the displacer.

Conduction through the cylinder wall for the various geometries is calculated using a wall thickness based on the maximum allowable stress for the stainless steel cylinder. This assures adequate wall strength and minimizes unnecessary conduction due to an over estimate of wall thickness. The wall stress in a pressurized cylinder is,

$$\sigma = \frac{PD}{2t} \text{ or } \quad t = \frac{PD}{2\,\sigma_{max}} \tag{23}$$

further assume σ_{max} = 7000 psi (conservative) and substituting into the conduction equation with the conduction length equal to regenerator length minus stroke the following equation is obtained

$$Ql_k = K_6 \; \frac{D^2 \; P_h}{L-S} \tag{24}$$

Conduction in the regenerator can be significant but these losses are accounted for in the thermal inefficiency developed by Ackerman[8]. The conduction in the displacer/piston will be neglected since the sole function of the displacer/piston is to separate the low temperature from the high temperature region and it is therefore designed for minimal conduction. If under some condition cylinder conduction became the dominant loss, the piston conduction assumption should be reviewed.

Shuttle Heat Transfer Loss

Although through design this loss has been eliminated its value will be calculated for a similar geometry system with shuttle heat transfer to illustrate the benefit gained by the annular regenerator design. Zimmerman[9] presents the following equation for shuttle heat transfer

$$Ql_s = \frac{k \times \pi \times D \times S^2 \times (T_a - T_c)}{5.4 \times C \times L} \tag{25}$$

This ideal equation can be correlated with real results by multiplying the predicted result by a correlation factor. For sinusoidal motion at 144 cycles per minute and 540°R to 140°R temperature difference this factor is 0.71.

Net Refrigeration

The net refrigeration or the available refrigeration is:

$$Q_{net} = QI - Ql_{\Delta p} - Ql_{tin} - Ql_k$$

COMPUTER PROGRAM

A digital computer program has been developed based on the equations previously presented. The program is run with the length and the capacity ratio of the regenerator varying, while the cycles per minute are held constant. The program is then executed at another cycle rate.

The program first computes the regenerator volume. Using this information it calculates the size of the expansion volume. Using the proper stroke to length ratio to eliminate shuttle heat transfer, the program computes the regenerator inner and outer diameter, completely defining the expander geometry. The pressure drop loss, conduction loss and shuttle heat transfer are then calculated. The regenerator inefficiency must be supplied for each geometry and speed before this loss can be computed. The regenerator inefficiency is a function of capacity ratio and regenerator NTU's. The program computes and prints the capacity ratio and NTU's and requests the regenerator inefficiency be entered before it completes a loop.

The program prints the ideal refrigeration, all of the losses and the net refrigeration.

Example of Program Use

Although the program would allow the pressures, temperatures, compressor flow, regenerator material, refrigeration gas, regenerator mesh, porosity and wire diameter and the period of fill and vent to be varied these parameters were held constant at typical values.

The following parameters were used to illustrate the use of the program for optimization of a cryogenic refrigerator.

PARAMETERS (Held Constant)

Pressure Supply - Ph, PSIA	315
Pressure Suction - Pi, PSIA	115
Temperature Ambient - Ta, °R	540
Temperature Cold End - Tc, °R	140
Compressor Flow - W, lbm/hr	1.0
Regenerator Material	copper
Gas	helium
Mesh Porosity	0.67
Mesh	200
% of cycle high pressure blow	40
% of cycle low pressure blow	40
Prandtl Number	0.688
Gas Conduction - hr-ft-°R	0.0637

PARAMETERS (Varied)

Speed - N, Cycles/Minute	50, 100, 200, 400, 600
Capacity Ratio - CR	4, 8, 12, 20, 30
Regenerator Length - L, inches	1, 2, 3, 4, 5, 6, 7, 8

This combination of parameters yields 200 expander designs with performance specifications from which a relative optimum can readily be chosen.

RESULTS

The results of the computer analysis can be used to graphically illustrate the characteristics of an external annular regenerator. This partial presentation of the output from the example can be used to both illustrate the magnitude of each loss versus the varied parameters and also to indicate what design geometry would yield a maximum refrigeration output for a given compressor flow.

The relative values of the losses which consume available refrigeration are illustrated by Figures 6 and 7. The regenerator inefficiency and cylinder conduction both rapidly decrease as the regenerator and cylinder are lengthened. The pressure drop loss increases as the length of the regenerator increases. The net refrigeration for the 50 CPM case (Figure 6) is significantly greater than for the 600 CPM case (Figure 7) for two reasons. First, the QI is significantly less ($\sim$1.5 watts) for the 600 CPM case; but secondly, and more important, the pressure drop loss and regenerator thermal inefficiency loss are significantly larger for the 600 CPM case. The conduction is almost negligible at the optimum point in both cases and the shuttle heat transfer has been eliminated. Shuttle heat transfer losses are shown here for reference. A study of the total losses including shuttle heat transfer illustrates the reason why systems designed with regenerators internal to the displacer/piston typically optimized at relatively higher speeds with shorter strokes and longer pistons, at the expense of greater pressure drop and regenerator inefficiency losses.

Figures 6 and 7 also tabulate the geometry of the expanders. The optimum geometry and performance specifications for the 50 CPM and 600 CPM cases with and without shuttle heat transfer is presented below for comparison.

Speed - CPM	50	600
Regenerator Length - in	5	4
Diameter of Displacer - in	1.33	0.52
Stroke - in	0.385	0.19
Net Refrigeration External Regenerator - watts/lbm helium	7.3	4.3
Net Refrigeration Internal Regenerator - watts/lbm helium	5.1	4.0

A comparison of net refrigeration with an external versus internal regenerator illustrates the benefit to be gained from the external regenerator design. The benefit at lower speeds is much more significant than at higher speeds because the lower speed system is physically larger and therefore has more potential shuttle losses which can be eliminated.

Figure 8 illustrates the effect of regenerator length at various capacity ratios on net refrigeration. Curves of this nature can be used to determine the optimum capacity ratio for a given speed expander.

Figure 9 illustrates the effect of speed at various capacity ratios on net refrigeration. This information can be used to determine the optimum capacity ratio and cycle rate for a thermally optimized machine.

Figure 10 illustrates Net Refrigeration Vs. Capacity Ratio at Constant Speed. The theoretical limit of refrigeration which could be produced if there were no losses and no regenerator void volume, and QI the maximum refrigeration which can be produced after consideration is given to regenerator void volume are shown for reference. The family of convex curves illustrates the effect of displacer speed on net refrigeration produced. Each particular curve displays the effect of capacity ratio at a given displacer speed. At lower capacity ratios the net refrigeration decreases because the losses rapidly increase; at high capacity ratios the net refrigeration decreases because the regenerator void volume consumes a large percent of the compressed gas yielding a smaller amount for refrigeration production.

The locus of maximums in Figure 10 indicates that there may be some benefit in net refrigeration to be gained by operating at speeds lower than 50 cycles/minute. This was not investigated because the benefit appears small, furthermore, a cycle rate of less than 50 CPM would yeild an intolerably large device with long cool-down time, and the more massive reciprocating displacer would have greater vibration. Therefore, although the program is designed to determine the thermal optimum a designer must introduce other restraints on the system design so that the overall package is acceptable.

CONCLUSIONS

An uncomplicated approach has been developed which can be used to thermally analyze a cryogenic refrigerator with a regenerator designed to eliminate shuttle heat transfer. In addition, a computer program has been developed and used to illustrate the technique and to estimate the relative merits of pursuing annular regenerator expanders.

The computer results indicate (1) a thermally more efficient expander can be manufactured using the annular external regenerator rather than an internal regenerator, (2) the most significant benefits are obtained on low speed devices, (3) with shuttle heat transfer eliminated, thermal inefficiency in the regenerator becomes the significant loss. This suggests that a still more efficient optimum could be obtained if pressure drop losses vs. thermal inefficiency loss were explored.

Experimental prototype test work and manufacturing techniques will have to be pursued before a final decision can be made on the usefulness of the annular regenerator, but the results of the thermal analysis indicate that further work on this configuration is warranted.

LIST OF SYMBOLS

C	Clearance
C_p	Specific heat at constant pressure
CR	Capacity ratio of regenerator
D	Diameter
D_h	Hydraulic diameter
dp	Differential pressure change
dx	Differential length change
ΔP	Pressure drop
ΔT	Change in temperature of a location in the regenerator per cycle
f	Friction factor
G	Mass flux
g_c	Gravitational constant
K	Constant
k	Thermal conductivity
L	Regenerator length
M	Mass
N	Cycles per minute of the displacer
NTU	Number of heat transfer units
N_{RE}	Reynolds Number
P	Pressure
p	Porosity
π	Pi
Q	Heat flow
QI	Ideal refrigeration

Ql	Refrigeration loss
ρ	Density
S	Stroke
σ	Wall stress
T	Temperature
t	Wall thickness
μ	Viscosity
V	Volume
W	Compressor flow

SUBSCRIPTS

a	Ambient
c	Cold
cu	Copper
Δp	Pressure drop
g	Gas
h	High pressure
he	Helium
i	Initial
k	Conduction
l	Low pressure
max	Maximum allowable
net	Net refrigeration
o	Exiting
r	Regenerator
s	Shuttle heat transfer
t	Thermal
tin	Thermal inefficiency

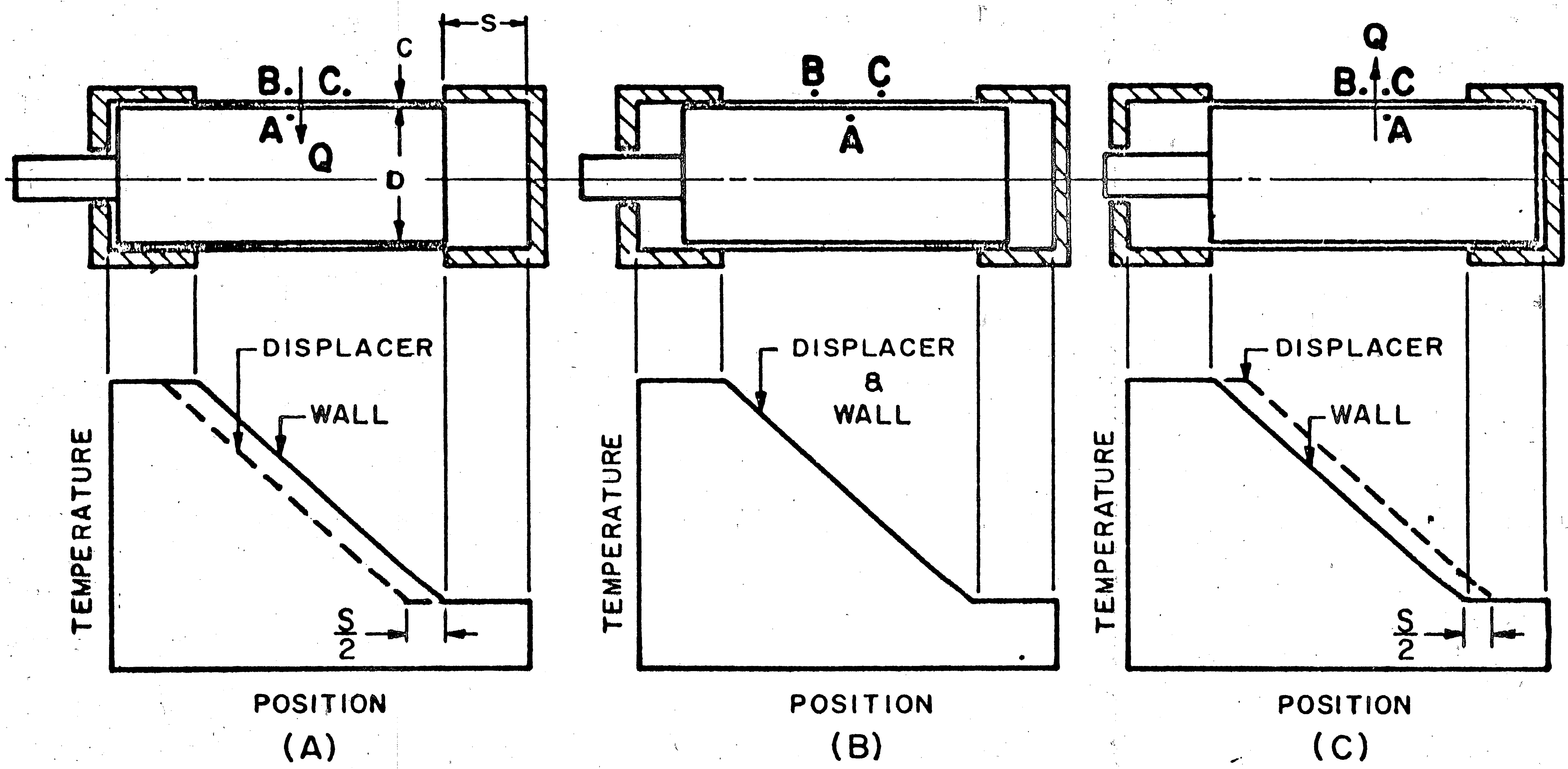

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FIGURE 1* SHUTTLE HEAT TRANSFER MECHANISM

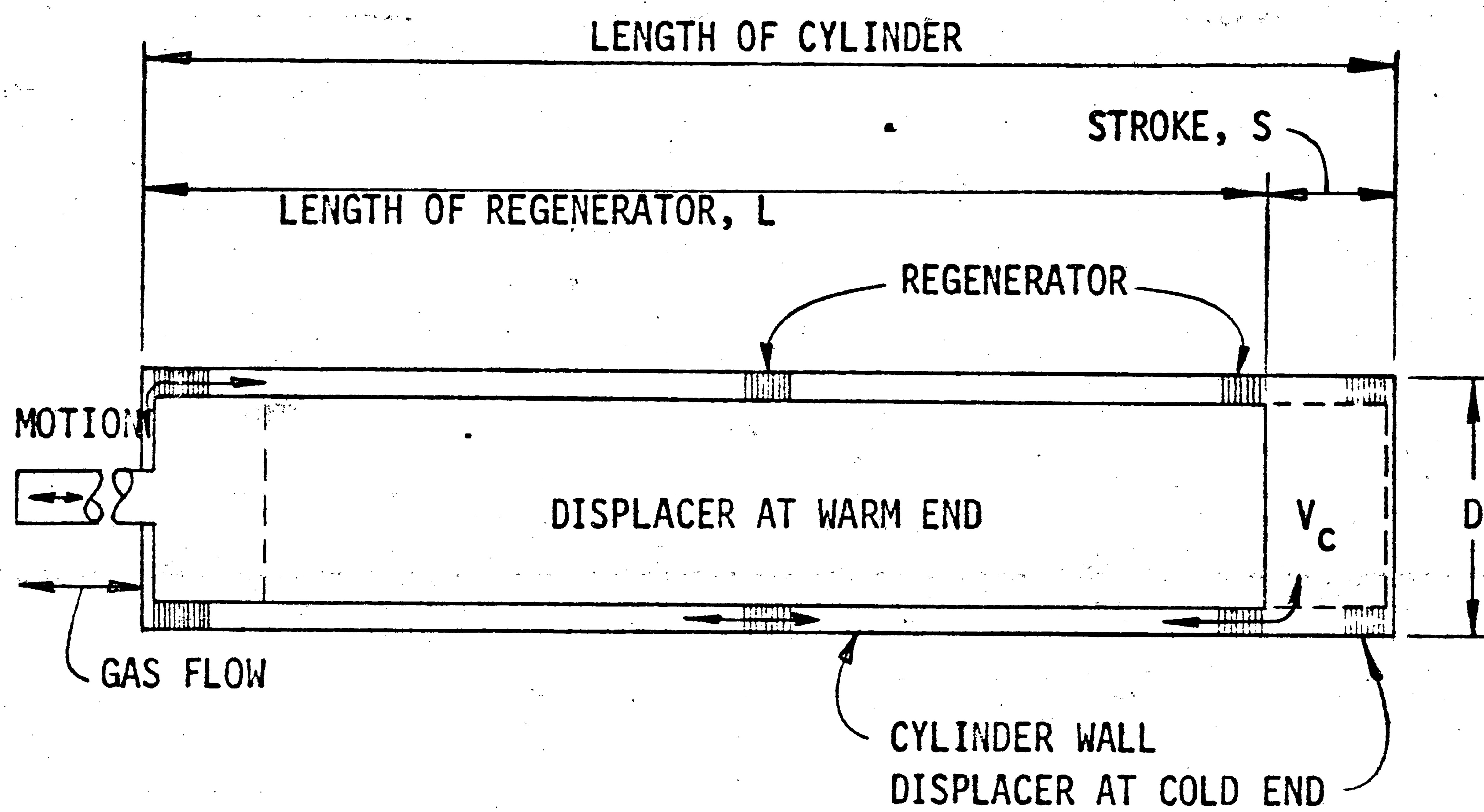


FIGURE 2 MODEL OF EXPANDER FOR NO SHUTTLE HEAT TRANSFER

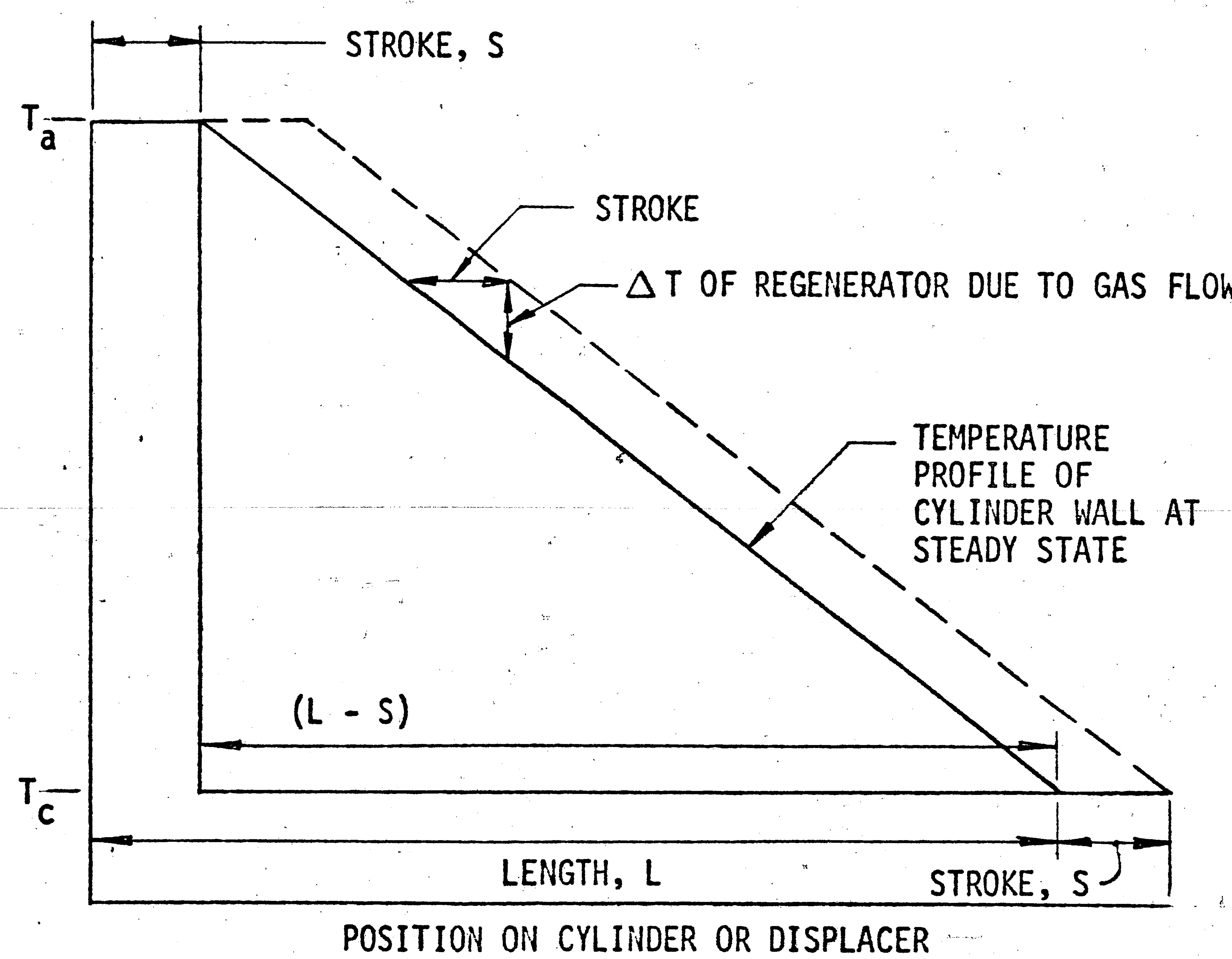
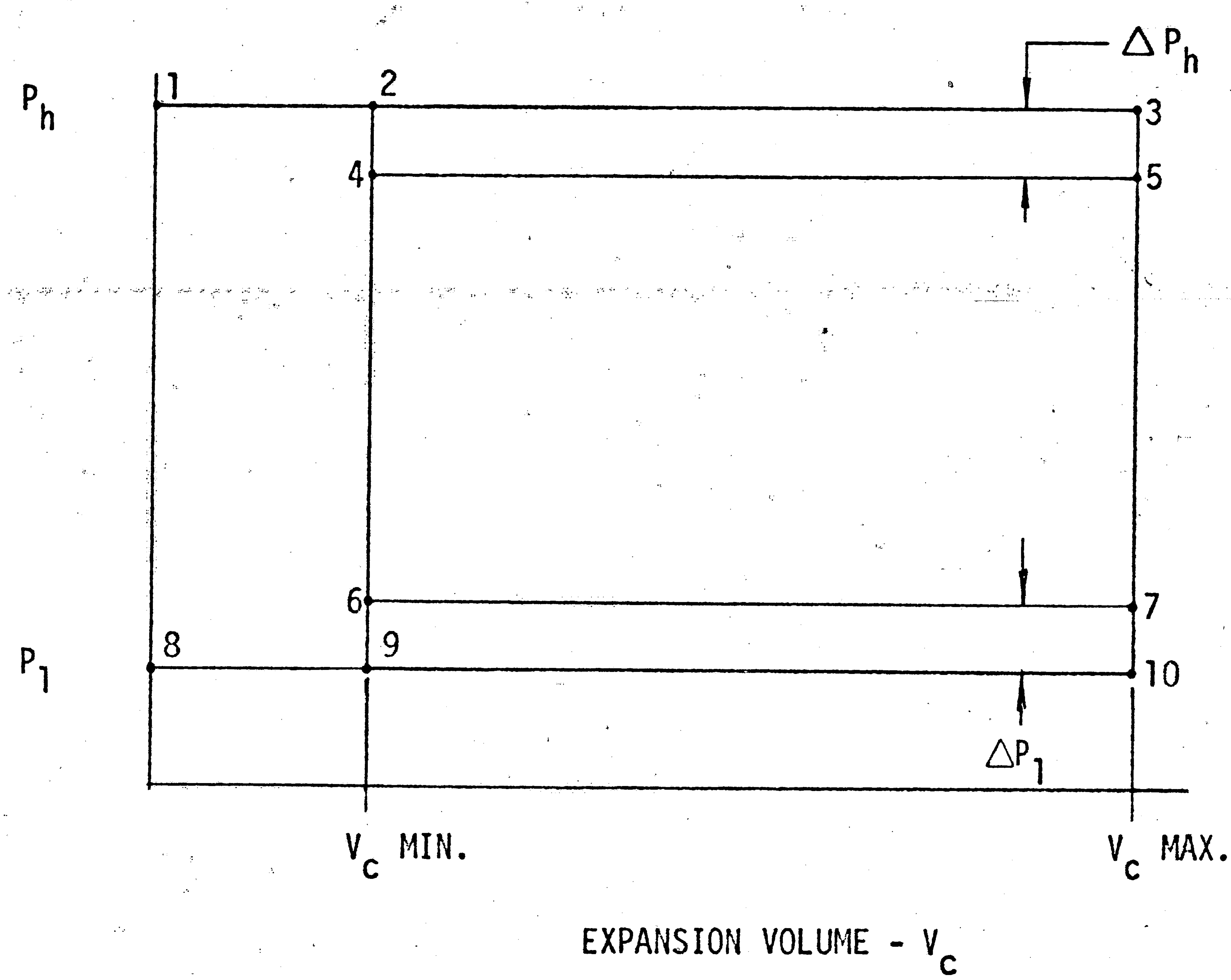


FIGURE 3 THERMAL PROFILE OF CYLINDER AND REGENERATOR



Q THEORETICAL LIMIT CYCLE: 1-3-10-8-1

Q IDEAL CYCLE: 2-3-10-9-2

Q IDEAL CYCLE - PRESSURE DROP LOSS: 4-5-7-6-4

FIGURE 4 P-V DIAGRAM FOR GIFFORD-McMAHON CYCLE

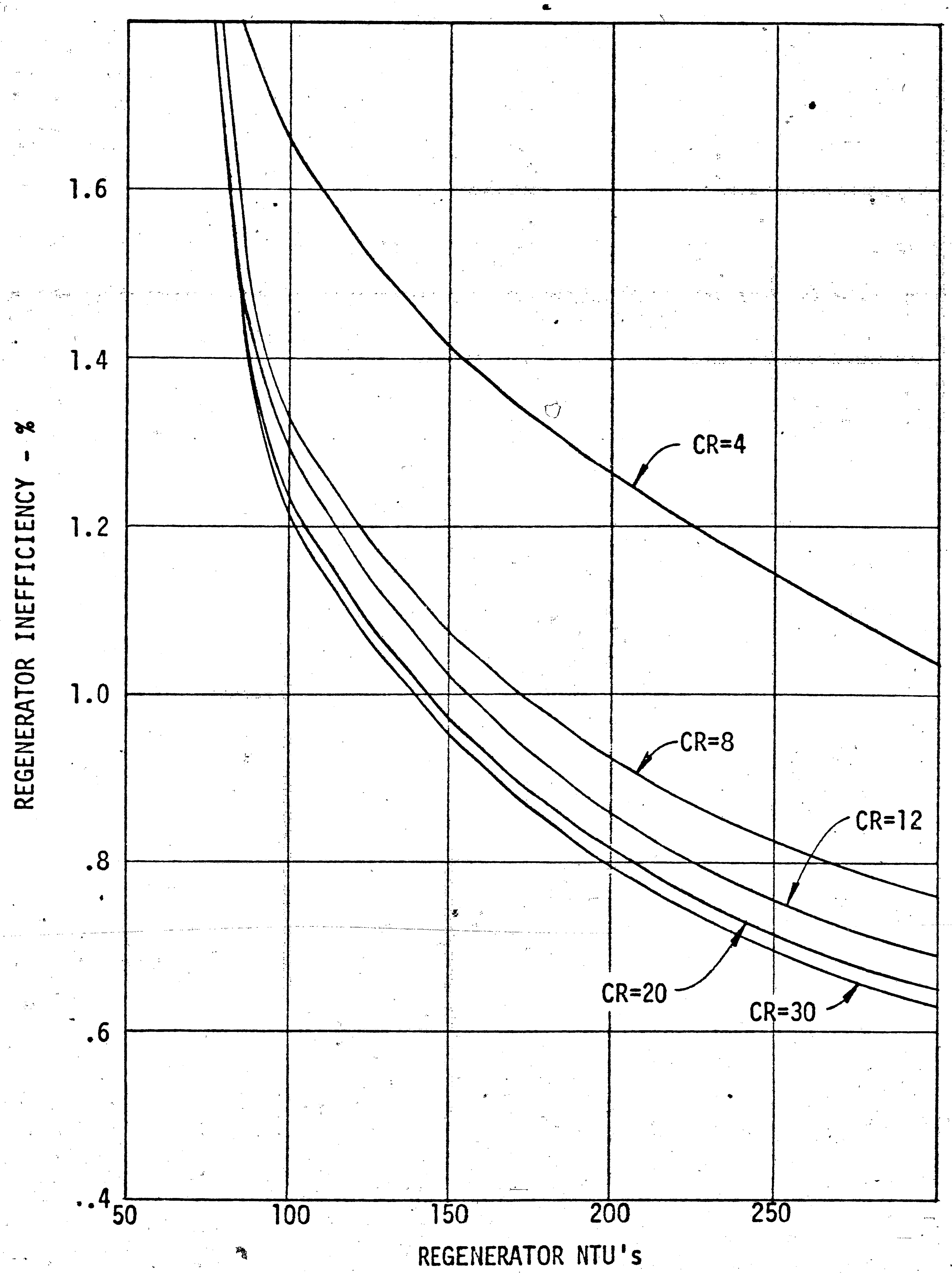


FIGURE 5 THERMAL INEFFICIENCY VS REGENERATOR NTU's

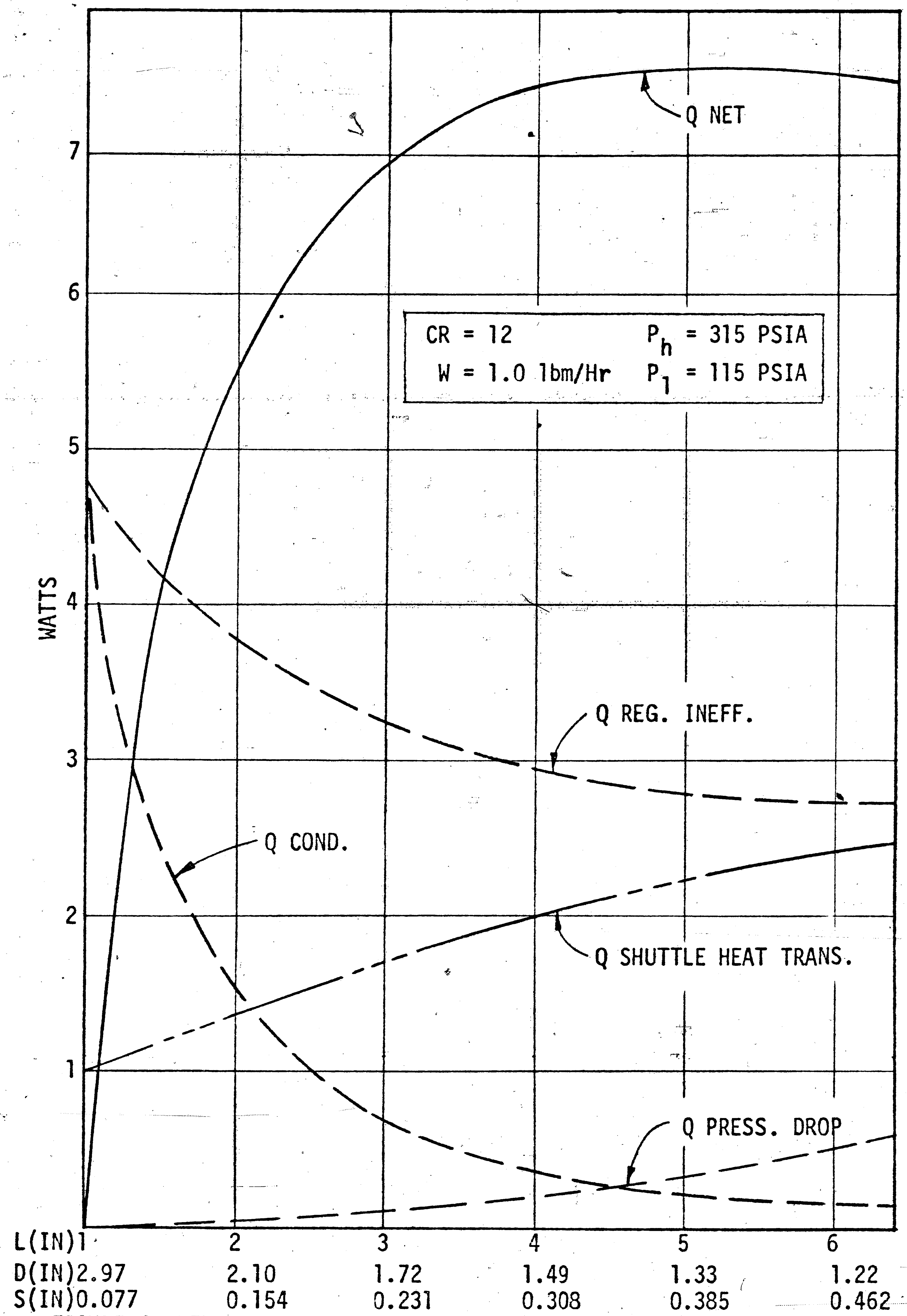
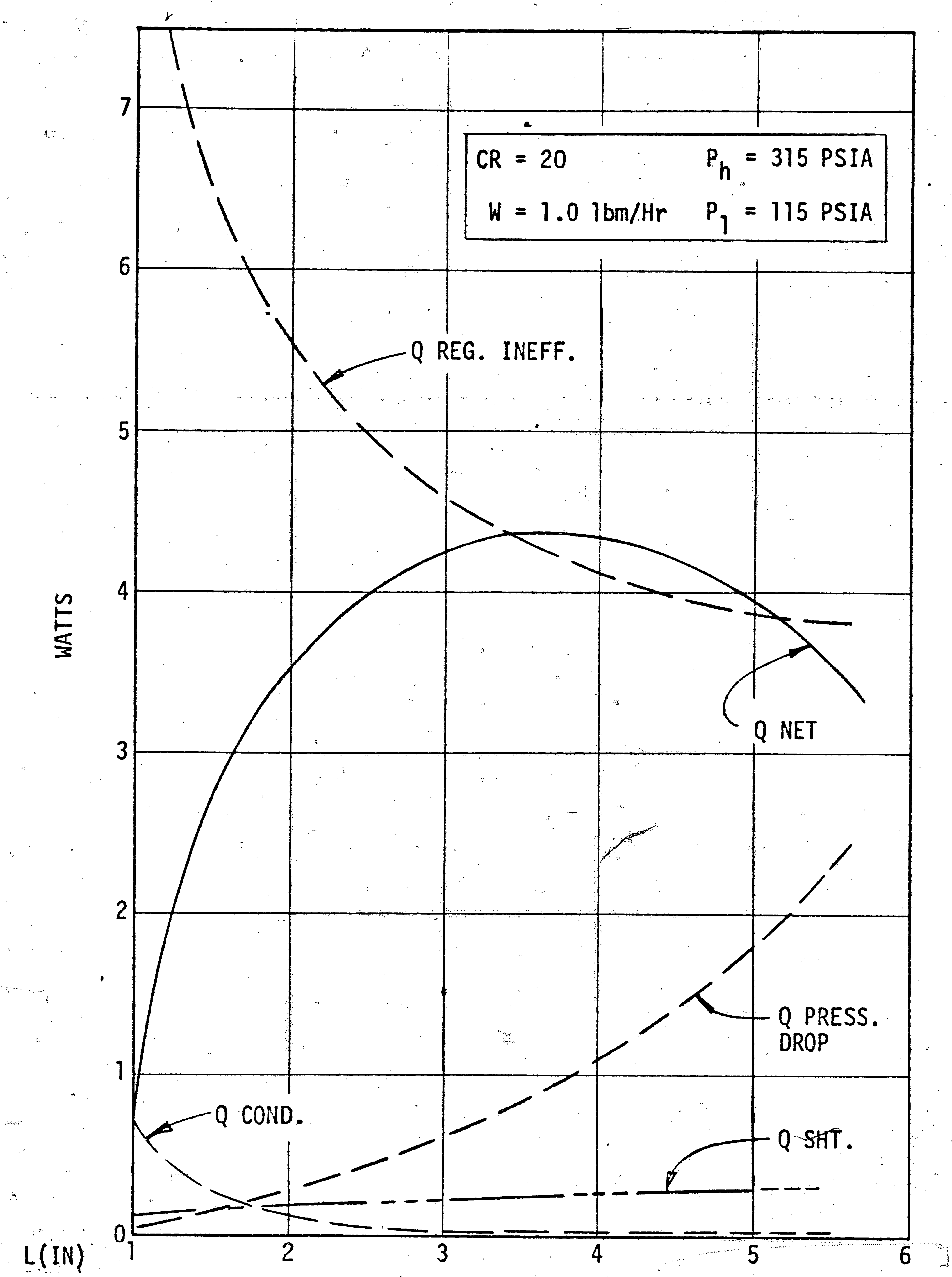


FIGURE 6 NET REFRIGERATION AND LOSSES VS REGENERATOR LENGTH AT MAXIMUM REFRIGERATION CAPACITY RATIO FOR 50 CPM SPEED



D(IN)	1.04	0.735	0.60	0.52	0.465
S(IN)	0.048	0.095	0.143	0.190	0.238

FIGURE 7 NET REFRIGERATION AND LOSSES VS REGENERATOR LENGTH AT MAXIMUM REFRIGERATION CAPACITY RATIO FOR 600 CPM SPEED

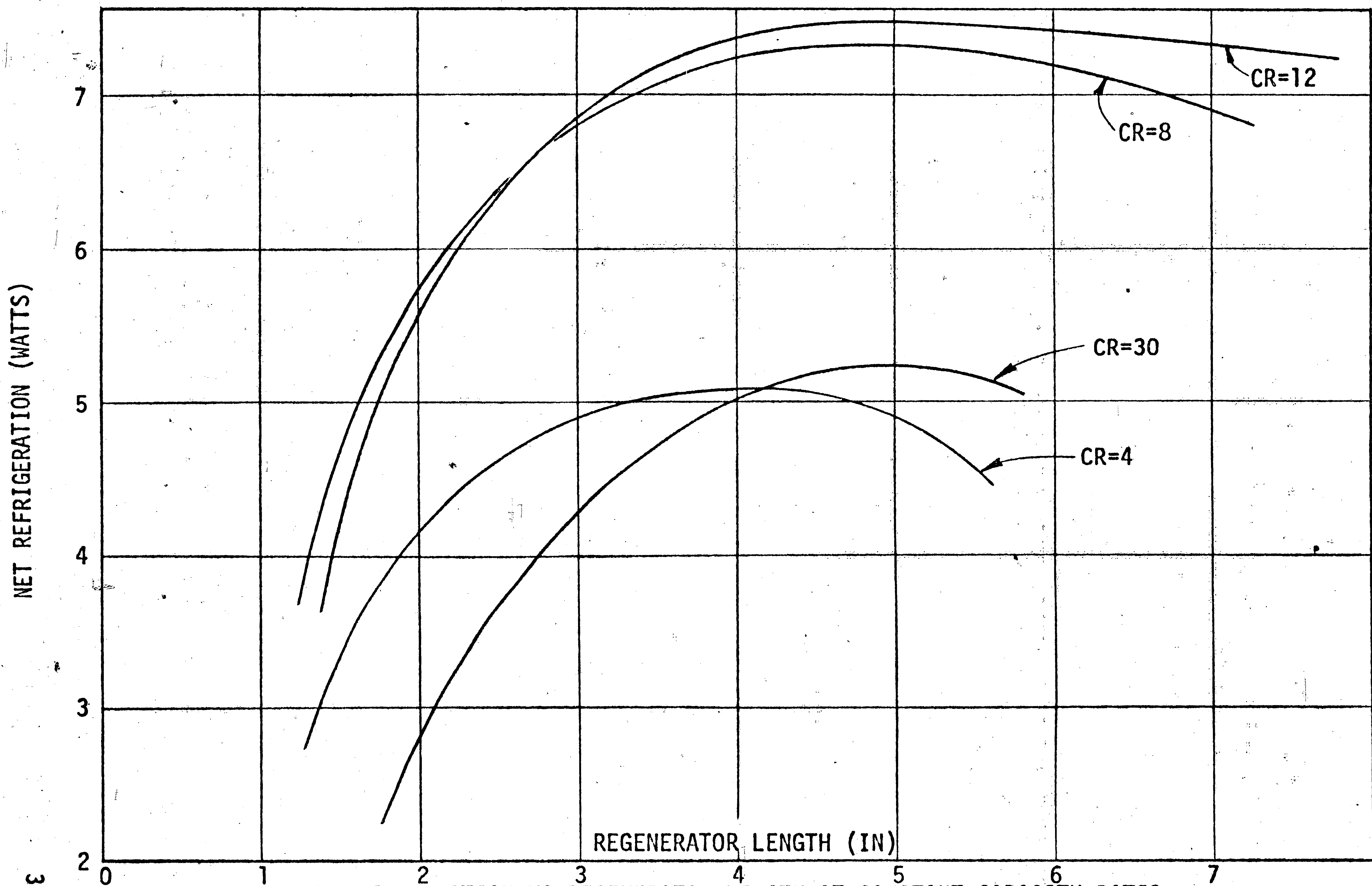


FIGURE 8 NET REFRIGERATION VS REGENERATOR LENGTH AT CONSTANT CAPACITY RATIO

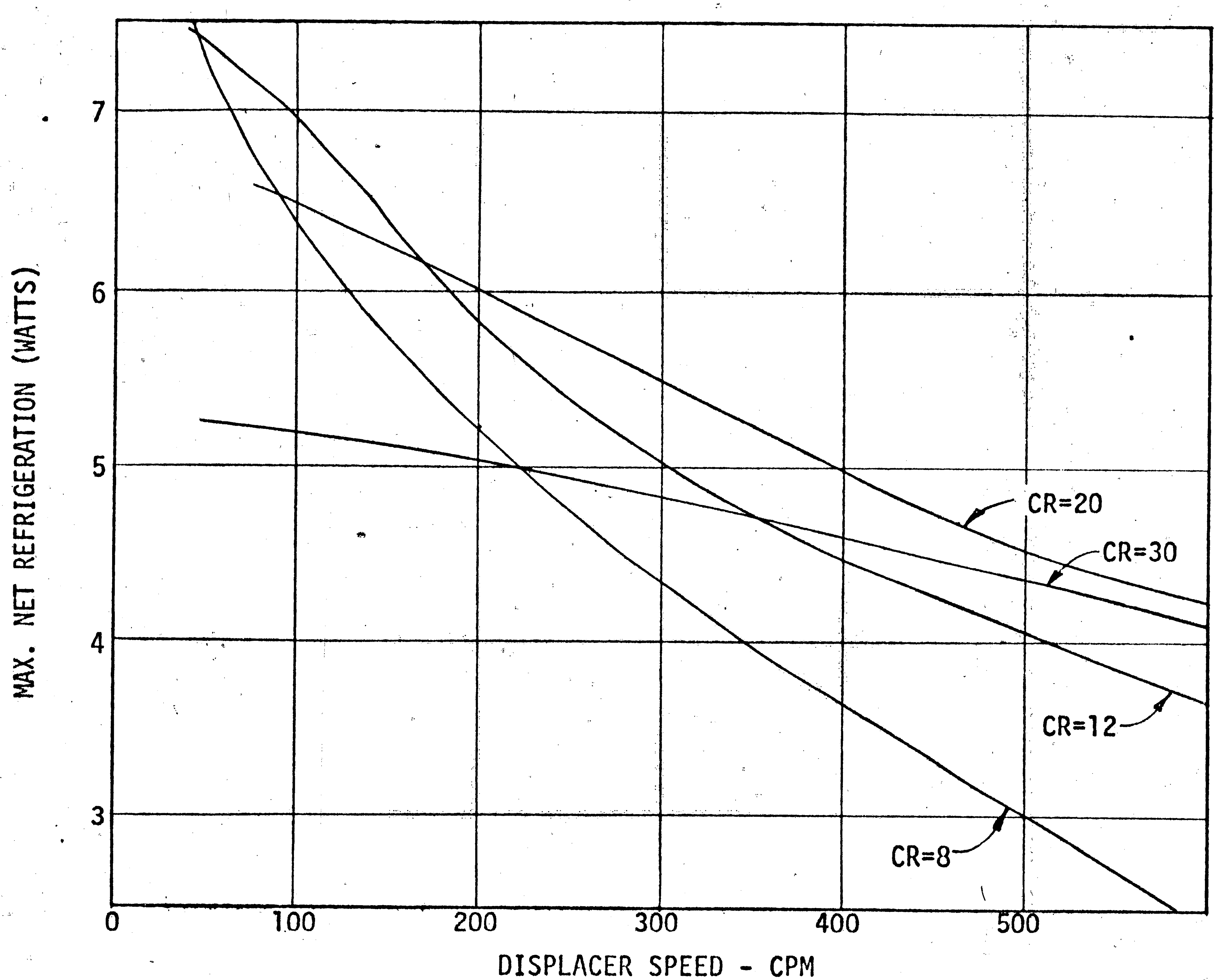


FIGURE 9 NET REFRIGERATION VS DISPLACER SPEED AT CONSTANT CAPACITY RATIO

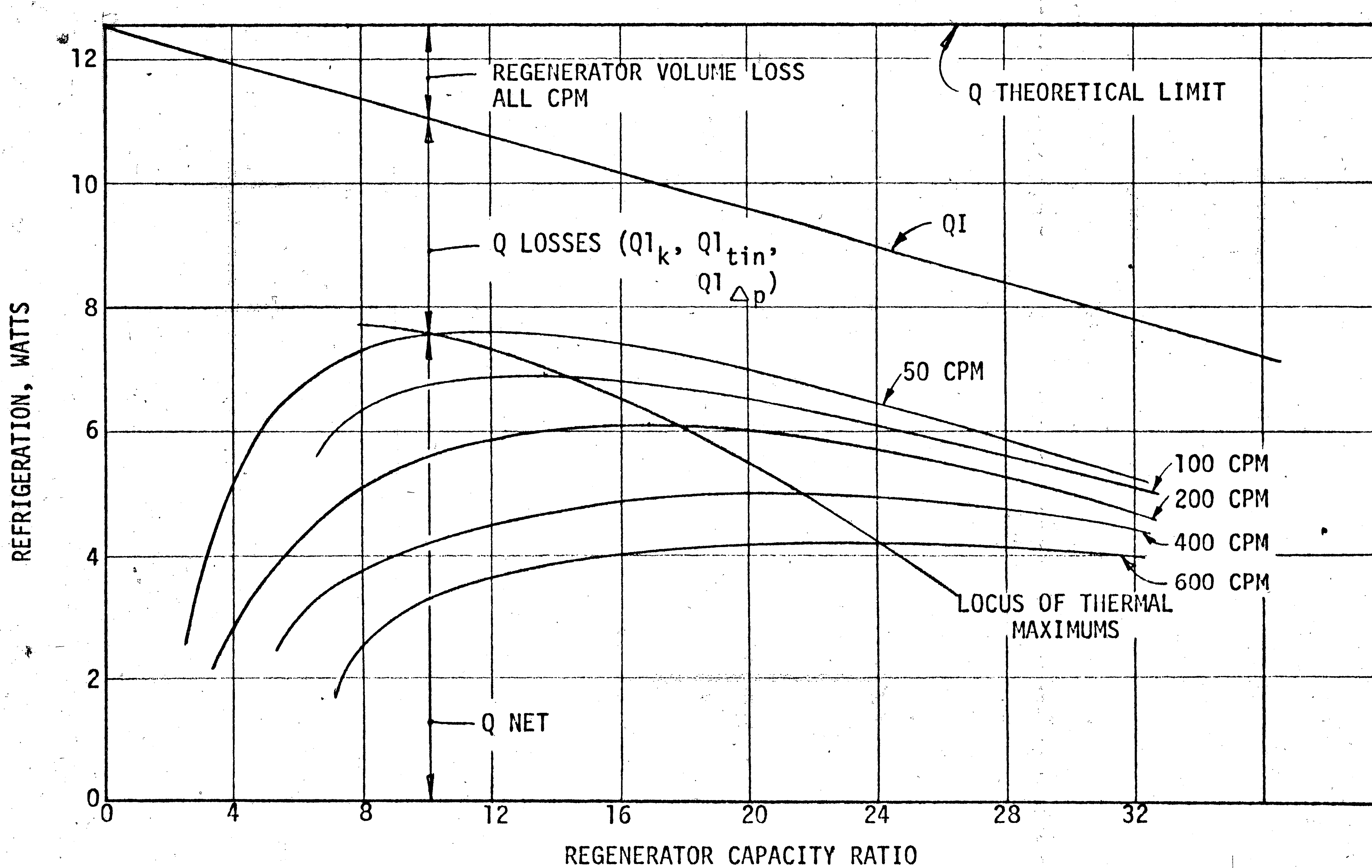


FIGURE 10 NET REFRIGERATION AND QI VS CAPACITY RATIO AND CONSTANT SPEED

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BIOGRAPHY

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He was born in Scranton, Pennsylvania on June 5, 1946, of John V. Galdieri, Sr. and Gertrude Yencho Galdieri. He obtained a Bachelor of Science Degree in Mechanical Engineering summa cum laude from Lehigh University in 1968.